Interacting Multiple Model Extended Kalman Filter for Tracking Target Executing Evasive Maneuvers

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Abstract:

A 3-model interacting multiple model extended Kalman filter with constant velocity, constant acceleration and constant turn models (IMMEKF-VAT) is proposed for tracking a maneuvering target undergoing acceleration as well as turn maneuvers in the Cartesian $X-Z$ plane. Its performance is compared with 2-model IMMEKF-VA. Performance comparison indicates that the IMMEKF-VAT provides smoother estimates of target states only during turn maneuver at the cost of extra computation. IMMEKF-VA should suffice for tracking targets executing constant acceleration as well as constant turn maneuvers.

Key Words: IMMEKF, Target tracking, Constant turn model, state estimation

I INTRODUCTION

Kalman filter which is used extensively in target tracking applications performs optimally when the model describing the target motion is specified correctly. In tracking applications, for targets moving with constant velocity (CV), the state model includes the first derivative of position and for targets moving with constant acceleration (CA) it includes second derivative of position [1,2]. Models with second order derivatives are preferred for tracking maneuvering targets and referred to as acceleration models [1]. A target undergoing coordinated turn requires a constant turn (CT) model to describe the target motion. However, tracking a randomly maneuvering target with highly time varying and uncertain dynamics requires an adaptive state estimation. Interacting Multiple Model (IMM) Kalman filter [1,3,4] is one such adaptive estimator which is based on the assumption that a finite number of models are required to characterize the target motion at all times. The IMMKF uses several target motion models (i.e. constant velocity, constant acceleration, co-ordinate turn model etc.) and has been successfully applied to track large maneuvering targets. IMMKF may use one model for straight and level flight and different models for maneuvers or turns. The IMMKF always maintains all of the models and blends their outputs with weights that are computed probabilistically. In addition to the state estimates for each motion model, the IMMKF maintains an estimate of the probability that the target is moving in accordance with each model. The CV and CA models are commonly used models in IMM filter and one can find many applications of 2 model-IMM filter with CV and CA models for tracking maneuvering targets [1]. This paper presents the 3 model-IMM with CV, CA and CT models to track a maneuvering target undergoing a turn maneuver. The paper gives the details of the CT model, its inclusion in the IMM filter and results of its performance using simulated data.

II. IMMEKF

The nonlinear relationship between the tracking model states and the radar measurements necessitates the inclusion of extended Kalman filter in IMM filter and hence the name IMMEKF. The architecture of the IMMEKF algorithm is shown in Fig-1. The description of the IMMEKF algorithm is presented in [1,3,4]. To bring out the effect of inclusion of CT model in IMM filter following two cases have been studied:

(i) 2 model-IMM Extended Kalman filter with CV and CA models (IMMEKF-VA)

(ii) 3 model-IMM Extended Kalman filter with CV, CA and CT models (IMMEKF-VAT)

The performance of both the algorithms has been evaluated and compared using simulated trajectory of a target moving at constant velocity undergoing a constant turn motion and constant acceleration motion in the Cartesian $X-Z$ plane as observed by radar/seeker. The CV, CA and CT models used in the IMMEKF-VA and IMMEKF-VAT are described below. Since the CT model has a different state representation compared to CV and CA models, a method to convert between the different state representations is also required [5, 6].

III. TARGET MOTION MODELS

Constant velocity (CV) model

The state vector consists of target position and velocity along x- and z-axis.

$$X_{CV} = \begin{bmatrix} x & \dot{x} & z & \dot{z} \end{bmatrix}^T \quad (1)$$

The state transition matrix is given in below:

$$\Phi_{CV} = \begin{bmatrix} F_{CV} & 0 \\ 0 & F_{CV} \end{bmatrix} \quad F_{CV} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \quad (2)$$

where T is the time interval of the data samples

In this model, the change in velocity (acceleration) is assumed to be Gaussian white noise with standard deviation $\sigma_{\ddot{x}}$ and a target maneuver time constant τ. The process noise matrix Q_{CV} would then be given by:

$$Q_{CV} = 2\sigma_{\ddot{x}}^2 \tau \begin{bmatrix} \varsigma_{CV} & 0 \\ 0 & \varsigma_{CV} \end{bmatrix} \qquad \varsigma_{CV} = \begin{bmatrix} \frac{T^3}{3} & \frac{T^2}{2} \\ \frac{T^2}{2} & T \end{bmatrix} \quad (3)$$

For radar/seeker measurements, the measurement model would be:

$$\rho = \sqrt{x^2 + z^2} \qquad \dot{\rho} = \frac{x\dot{x} + z\dot{z}}{\sqrt{x^2 + z^2}}$$

$$\theta = \tan^{-1}\left(\frac{z}{x}\right) \qquad \dot{\theta} = \frac{x\dot{z} - z\dot{x}}{x^2 + z^2} \tag{4}$$

where ρ = range (m), $\dot{\rho}$ = range rate (m/sec), θ = azimuth (rad) and $\dot{\theta}$ = azimuth rate (rad/sec)

Constant Acceleration (CA) Model

The state vector consists of target position, velocity and acceleration along x- and z-axis.

$$X_{CA} = [x \quad \dot{x} \quad \ddot{x} \quad z \quad \dot{z} \quad \ddot{z}]^T \tag{5}$$

The state transition matrix is given below:

$$\Phi_{CA} = \begin{bmatrix} F_{CA} & 0 \\ 0 & F_{CA} \end{bmatrix} \qquad F_{CA} = \begin{bmatrix} 1 & T & \frac{T^2}{2} \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} \tag{6}$$

In this model, change in acceleration (jerk) is assumed to be Gaussian white noise with standard deviation $\sigma_{\ddot{x}}$ and target maneuver time constant τ. The process noise matrix Q_{CA} would then be given by:

$$Q_{CA} = 2\sigma_{\ddot{x}}^2 \tau \begin{bmatrix} \varsigma_{CA} & 0 \\ 0 & \varsigma_{CA} \end{bmatrix} \qquad \varsigma_{CA} = \begin{bmatrix} \frac{T^5}{5} & \frac{T^4}{4} & \frac{T^3}{3} \\ \frac{T^4}{4} & \frac{T^3}{3} & \frac{T^2}{2} \\ \frac{T^3}{3} & \frac{T^2}{2} & T \end{bmatrix} \tag{7}$$

The measurement model would be the same as eq. (4).

Constant turn (CT) model

The CT model assumes that the target is moving along a circular path with constant speed and turning rate. Changes in speed and turning rate are modeled as Gaussian white noise. Best results using CT model will be accomplished with polar representation of the velocity in the state vector [5-7]. This polar system should be centered at the target and the state vector includes target speed, angular direction and turning rate. Therefore, the state vector for the CT model would be:

$$X_{CT} = [x \quad z \quad V \quad \alpha \quad \omega]^{\mathrm{T}} \tag{8}$$

In this state vector x and z represent the target position in a Cartesian coordinate system, V is the resultant velocity in the xz plane, α is the angular direction and ω is the turning rate ($\dot{\alpha}$). The discrete representation of the CT model is given below:

$$X(k+T) = f_k(X(k)) = X(k) + \begin{bmatrix} SW\, v_x - CW\, v_z \\ SW\, v_x + CW\, v_z \\ 0 \\ \omega \\ 0 \end{bmatrix} T \tag{9}$$

where $v_x = V\cos(\alpha) \qquad v_z = V\sin(\alpha)$

$$SW = \frac{\sin(\omega T)}{\omega T} \qquad CW = \frac{1-\cos(\omega T)}{\omega T}$$

The transition matrix for CT model required for Kalman filter covariance matrix propagation is computed as:

$$\Phi_{CT} = \begin{bmatrix} 1 & 0 & F_{13} & F_{14} & F_{15} \\ 0 & 1 & F_{23} & F_{24} & F_{25} \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & T \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{10}$$

where

$$F_{13} = T(SW\cos(\alpha) - CW\sin(\alpha))$$
$$F_{14} = -T(SW\, v_z + CW\, v_x)$$
$$F_{15} = T(AW\, v_x - BW\, v_z)$$
$$F_{23} = T(SW\sin(\alpha) + CW\cos(\alpha))$$
$$F_{24} = T(SW\, v_x - CW\, v_z)$$
$$F_{25} = T(AW\, v_z + BW\, v_x)$$
$$BW = \frac{\partial CW}{\partial \omega} = \frac{1}{\omega}\left(\sin(\omega T) - \frac{1-\cos(\omega T)}{\omega T}\right)$$
$$AW = \frac{\partial SW}{\partial \omega} = \frac{1}{\omega}\left(\cos(\omega T) - \frac{\sin(\omega T)}{\omega T}\right)$$

The process noise covariance matrix representing white process noise that enters through the state ω and V is:

$$Q_{CT} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & T\sigma_{\dot{V}}^2 & 0 & 0 \\ 0 & 0 & 0 & \frac{T^3}{3}\sigma_{\dot{\omega}}^2 & \frac{T^2}{2}\sigma_{\dot{\omega}}^2 \\ 0 & 0 & 0 & \frac{T^2}{2}\sigma_{\dot{\omega}}^2 & T\sigma_{\dot{\omega}}^2 \end{bmatrix} \tag{11}$$

where $\sigma_{\dot{\omega}}$ and $\sigma_{\dot{V}}$ represent the standard deviations of the change in the turn rate and velocity respectively. In this case measurement model would be:

$$\rho = \sqrt{x^2 + z^2} \qquad \dot{\rho} = \frac{xv_x + zv_z}{\sqrt{x^2 + z^2}}$$

$$\theta = \tan^{-1}\left(\frac{z}{x}\right) \qquad \dot{\theta} = \frac{xv_z - zv_x}{x^2 + z^2} \tag{12}$$

State Conversion *[5,6]*

IMMEKF algorithm expects the state vector representation to be the same in all models. The states of all models should be transformed to a common set of states to perform state and covariance mixing in IMMEKF. In this paper, since the state estimation is carried out in Cartesian coordinate system, the state vector used in the CA model (eq. 5) is chosen as the common reference set. CV model state vector with a size (4x1) (eq. 1) could be extended to (6x1) with acceleration states set identically zero. The (5x1) state vector of CT model (eq. 8) is in polar frame and it is transformed to (6x1) state vector of Cartesian frame as:

$$X_{TC} = [x \quad \dot{x}_C \quad \ddot{x}_C \quad z \quad \dot{z}_C \quad \ddot{z}_C] \tag{13}$$

where $\dot{x}_C = V\cos(\alpha) \qquad \dot{z}_C = V\sin(\alpha)$

$\ddot{x}_C = -\omega V\sin(\alpha) \qquad \ddot{z}_C = \omega V\cos(\alpha)$

After mixing and estimation, the Cartesian state vector X_{TC} has to be transformed back to the polar state vector X_{CT} (eq. 8).

where $V=\sqrt{\dot{x}_C^2+\dot{z}_C^2}$ $\quad \alpha=\tan^{-1}\left(\frac{\dot{z}_C}{\dot{x}_C}\right)$

$$\omega=\frac{\dot{x}_C\ddot{z}_C-\dot{z}_C\ddot{x}_C}{V^2} \qquad (14)$$

Similarly, polar form of covariance matrix P_{CT} of CT model has to be transformed to Cartesian frame as:

$$P_{TC}=A_{TC}P_{CT}A_{TC}^T \qquad (15)$$

where A_{TC} is transformation matrix given by the appropriate partial derivatives

$$A_{TC}=\frac{\partial X_{TC}}{\partial X_{CT}}=\begin{bmatrix}1 & 0 & 0 & 0 & 0\\ 0 & 0 & -\dot{z}_C & 0 & \cos(\alpha)\\ 0 & 0 & -\ddot{z}_C & -\dot{z}_C & -\omega\sin(\alpha)\\ 0 & 1 & 0 & 0 & 0\\ 0 & 0 & \dot{x}_C & 0 & \sin(\alpha)\\ 0 & 0 & \ddot{x}_C & \dot{x}_C & \omega\cos(\alpha)\end{bmatrix}$$

After estimation, the transformed covariance matrix P_{TC} has to be transformed back to polar frame as:

$$P_{CT}=A_{CT}P_{TC}A_{CT}^T \qquad (16)$$

where A_{CT} is the transformation matrix derived using partial derivatives

$$A_{CT}=\begin{bmatrix}1 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0\\ 0 & \frac{-\dot{z}_C}{V^2} & 0 & 0 & \frac{\dot{x}_C}{V^2} & 0\\ 0 & a_{42} & a_{43} & 0 & a_{45} & a_{46}\\ 0 & \frac{\dot{x}_C}{V} & 0 & 0 & \frac{\dot{z}_C}{V} & 0\end{bmatrix}$$

$$a_{42}=\frac{\ddot{z}_C(\dot{z}_C^2-\dot{x}_C^2)+2\dot{x}_C\dot{z}_C\ddot{x}_C}{V^4} \qquad a_{43}=\frac{-\dot{z}_C}{V^2}$$

$$a_{45}=\frac{\ddot{x}_C(\dot{z}_C^2-\dot{x}_C^2)-2\dot{x}_C\dot{z}_C\ddot{z}_C}{V^4} \qquad a_{46}=\frac{\dot{z}_C}{V^2}$$

IV. DATA SIMULATION

The maneuvering target motion trajectory is simulated using the constant velocity, constant acceleration and constant turn models for a total period of 18.75 seconds with sampling interval of 0.025 sec. The radar data tracking this maneuvering target is then generated. This data set is chosen to evaluate tracking as well as mode switching ability of IMMEKF-VAT. The data simulation is carried out with the following parameters:

- The target starts at position (1500, 1500) and moves with constant velocity of 100m/s in X-axis and -300 m/s in Z-axis
- First 100 samples (2.5 sec.) are generated using CV model
- Next 200 samples (5 sec.) are simulated using CT model with turn rate of 0.1 rad / sec
- Next 200 samples (5 sec.) are generated using CV model
- Next 150 samples (3.75 sec.) are generated using CA model with constant acceleration of 6g in both X- and Z-axes
- Last 100 samples (2.5sec.) are generated using CV model

Fig-2a and 2b show the simulated trajectories of position, velocity and acceleration in the X-axis and Z-axis respectively. The figures also indicate the modes that are dominant in each of the axes. This simulated data is used to generate measurements in polar coordinates. Random noise with the following variance is added to range, range rate, elevation, and elevation rate, to generate noisy measurement data. Measurement noise variances are:

for range = 2500 m^2
for range rate = 625 m/sec^2
for elevation = 0.25 $deg.^2$
for elevation rate = 9 $(deg./sec)^2$

Fig-2c shows the simulated measurements of range, range rate, elevation and elevation rate.

V. RESULTS AND DISCUSSION

In order to see the effect of inclusion of CT model, the performance of IMMEKF-VAT is compared with that of IMMEKF-VA. The algorithms are evaluated for their tracking performance, accuracy and consistency. Estimated mode probabilities from IMMEKF in each case are plotted to verify maneuver detection abilities of the algorithm.

Algorithm Performance Evaluation criteria are listed below

i) The percentage fit error (PFE) in X and Z positions, velocities and accelerations:

$$\text{PFE}=100*\frac{norm(X_t-\hat{X})}{norm(X_t)} \qquad (17)$$

where X_t represents the true value of the state $\hat{X}$ represents the estimated value of the same state

ii) Filter residual ($Z_m-\tilde{Y}$) plotted with the theoretical bounds

$$\pm 2\sqrt{S_{(k,k)}} \qquad (18)$$

where Z_m is the measured observation, $\tilde{Y}$ is the predicted observation and $S=(H\tilde{P}H^T+R)^{-1}$ is the innovation variance

iii) Root mean square position error:

$$\text{RMSPE}=\sqrt{\frac{1}{N}\sum_{i=1}^{N}\frac{(x_i-\hat{x}_i)^2+(z_i-\hat{z}_i)^2}{2}} \qquad (19)$$

where x_i, z_i are the true values of the position states in X and Z axis, $\hat{x}_i, \hat{z}_i$ are the estimated position states in X and Z axis

Similar computations for root mean square velocity error (RMSVE) and root mean square acceleration error (RMSAE) can be used

iv) Root sum square position error:

$$\text{RSSPE}(k)=\sqrt{(x(k)-\hat{x}(k))^2+(z(k)-\hat{z}(k))^2} \qquad (20)$$

Similar computations for root sum square velocity error (RSSVE) and root sum square acceleration error (RSSAE) can be used

v) The state error ($X-\hat{X}$) is plotted with the theoretical bounds of $\pm 2\sqrt{\hat{P}_{(k,k)}}$ (21)

vi) Estimated mode probability which indicates the transition from one model to the other

Tracker Initialization

Since the radar measurement is simulated, the initial state estimate is chosen as:

$$\hat{X}_{0j}=0.95*X_{0t} \qquad (22)$$

where $\hat{X}_{0j}$: Initial value of the states for the j[th] model and

X_{0t} : Initial true value of the states

The state error covariance matrix is initialized to:

$$\hat{P}_{0j} = diag\left[\left(X_{0t} - \hat{X}_{oj}\right)^2\right] \quad (23)$$

In case of 2 model IMMEKF-VA, the Markov chain transition matrix is chosen as:

$$p_{ij} = \begin{bmatrix} 0.999 & 0.001 \\ 0.001 & 0.999 \end{bmatrix} \quad (24)$$

and the initial mode probabilities $\mu = [\mu_{CV} \quad \mu_{CA}] = [0.9 \quad 0.1]$. In case of 3 model IMMEKF-VAT, the Markov chain transition matrix is chosen as:

$$p_{ij} = \begin{bmatrix} 0.999 & 0.001 & 0.001 \\ 0.001 & 0.999 & 0.001 \\ 0.001 & 0.001 & 0.999 \end{bmatrix} \quad (25)$$

and the initial mode probabilities $\mu = [\mu_{CV} \quad \mu_{CA} \quad \mu_{CT}] = [0.9 \quad 0.05 \quad 0.05]$.

Performance of IMMEKF Algorithms

The results presented here are average from fifty Monte Carlo simulations. Fig-3 show the comparison of estimated acceleration states from both the IMMEKF-VA and IMMEKF-VAT. During constant turn, though the estimates from IMMEKF-VA are good, it is not as smooth as the estimates from IMMEKF-VAT. This indicates that the CT model provides smoother estimates of the target states when the target performs turning maneuver. Thus the performance of IMMEKF-VAT is relatively better than IMMEKF-VA only during the turn phase of the target maneuver. Fig-4 shows the RSSPE, RSSVE and RSSAE (average from fifty Monte Carlo simulations). Here again errors in the estimates are relatively less in IMMEKF-VAT only during the turn maneuver phase. Similarly Fig-5 shows the acceleration state errors plotted with their theoretical bounds (average from fifty Monte Carlo simulations). The magnitudes of the bounds are relatively low in IMMEKF-VAT which shows the uncertainty in the state estimation is less. Fig-6a to 6b show the average mode probabilities computed from IMMEKF-VA and IMMEKF-VAT algorithms respectively which clearly reflects the mode switching between CV, CA and CT models.

More statistics on the performance of both the algorithms (average from fifty Monte Carlo simulations) are given Table 1 which shows that IMMEKF-VAT performed relatively better at the cost of higher execution time.

VI. CONCLUDING REMARKS

The performance evaluation of different IMMEKF algorithms viz. 2-model IMMEKF-VA and 3-model IMMEKF-VAT in tracking target executing both turning and acceleration maneuver is carried out. The results indicate that CA model is more important to be included in the IMM filter for tracking maneuvering target than the CT model. CT model provides smoother estimates of the target states only when the target performs constant turn maneuver. The performance of IMMEKF-VAT is relatively better than IMMEKF-VA in terms of all the evaluated performance criteria.

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VPS Naidu was born in Andhra Pradesh. He obtained M.E. in Medical electronics from Anna University Chennai in 1997. He is working at National Aerospace Laboratories, Bangalore as scientist since December 2001. His areas of interest are: image registration, tracking and fusion.

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Dr. J. R. Raol was born in 1947 in Gujarat. He obtained BE and ME degrees from MS university of Baroda in 1971 & 1974 and PhD from McMaster University, Canada in 1986. Worked in National Aerospace laboratories, Bangalore from 1975 to 1981 and was actively involved in the multidisciplinary control group's activities on human pilot modelling in fixed- and motion-based research simulators. He re-joined NAL in 1986 and is Head of the Flight Mechanics and Control Division of NAL. Current activities include modeling, identification, multisensor data fusion, fuzzy systems, genetic algorithms and neural networks. He is a senior member of the IEEE (USA), Fellow of IEE, Fellow of the Aeronautical Society of India, and Life Member of System Society of India. He has authored (jointly with Dr. Giriga G. and Dr. Jatinder Singh) a book on "Modeling and Parameter Estimation of Dynamic System", published by IEE, UK in 2004. He has 100 publications to his credit.

TABLE 1

PERCENTAGE FIT ERROR AND ROOT MEAN SQUARE ERRORS

Tracking filter	Percentage Fit Error				RMSPE	RMSVE	RMSAE	Execution time
	x-pos.	z-pos.	x-vel.	z-vel.				
IMM-VA	0.21	0.31	7.8	13.6	29.78	27.21	21.79	0.8 sec.
IMM-VAT	0.19	0.28	5.66	10.06	24.21	17.58	16.26	1.26 sec.

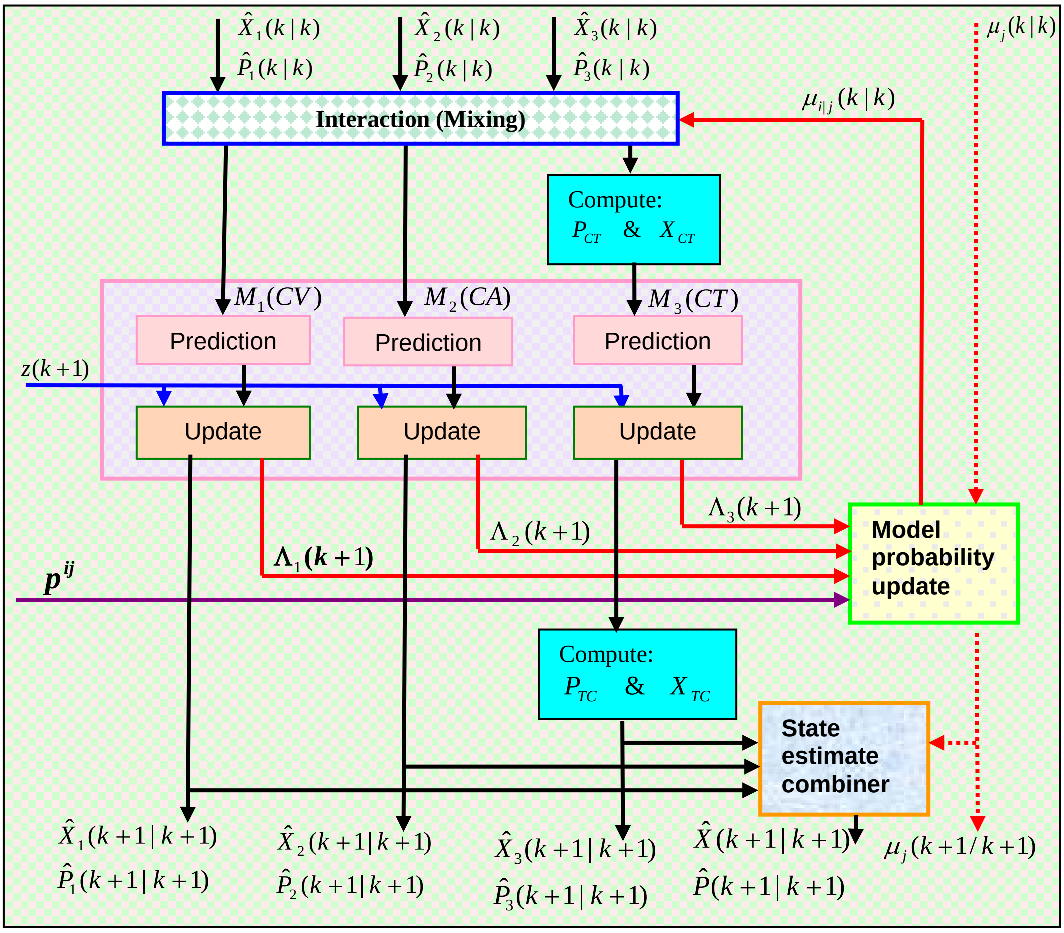

Fig-1. Information flow diagram of IMMEKF-VAT

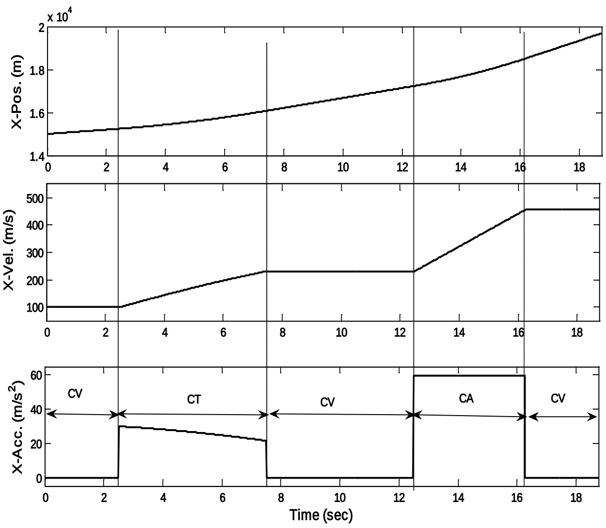

Fig-2a. Simulated X-axis data

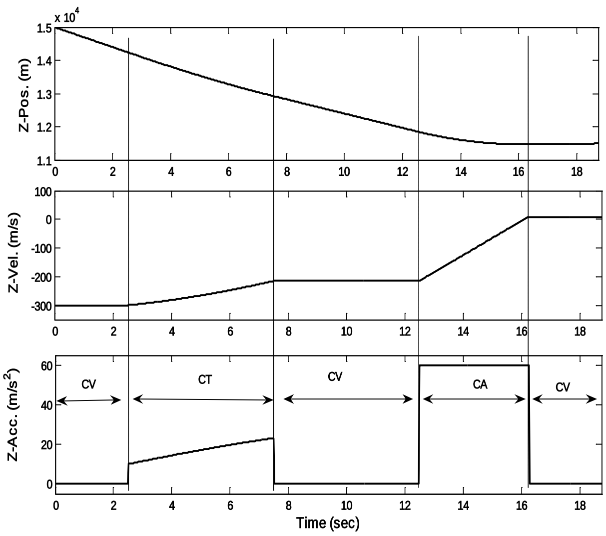

Fig-2b. Simulated Z-axis data

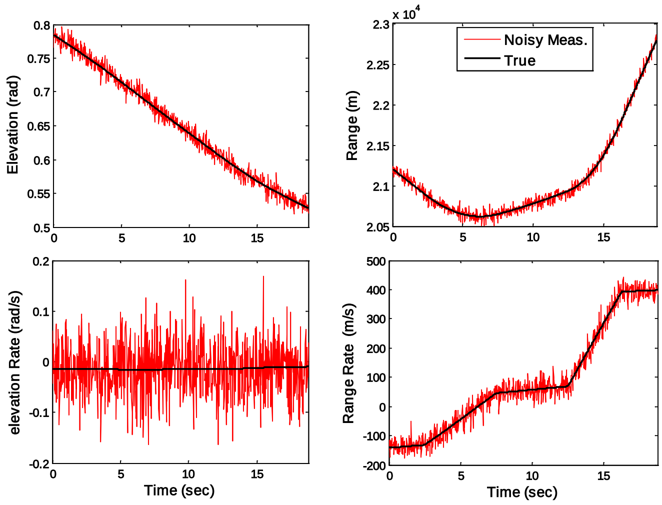

Fig-2c. Simulated radar measurements

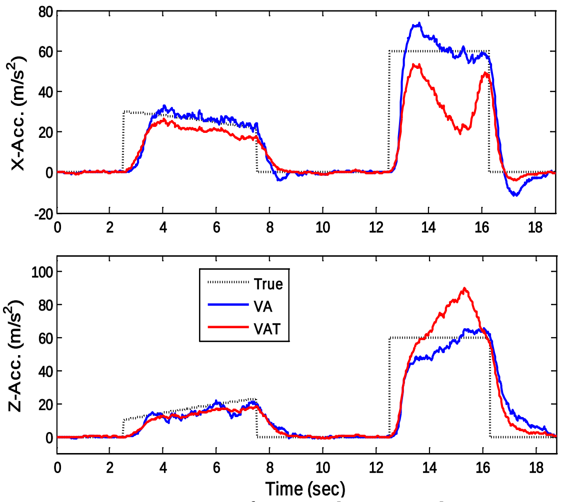

Fig-3. Comparison of estimated target acceleration

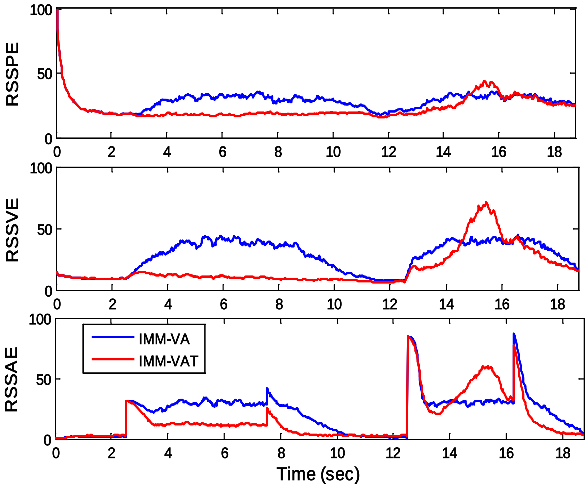

Fig-4. RSSE in estimation of target position, velocity and acceleration

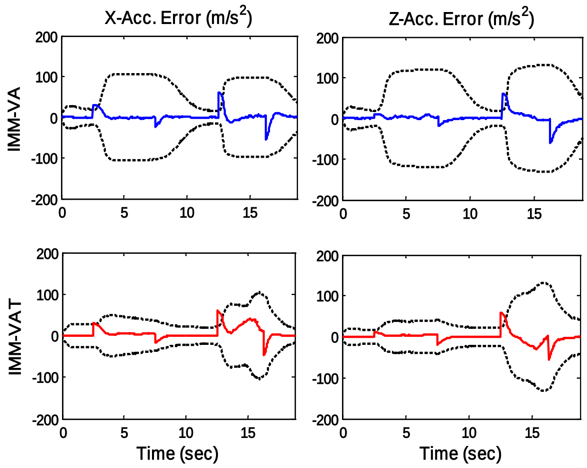

Fig-5. Acceleration state errors with bounds

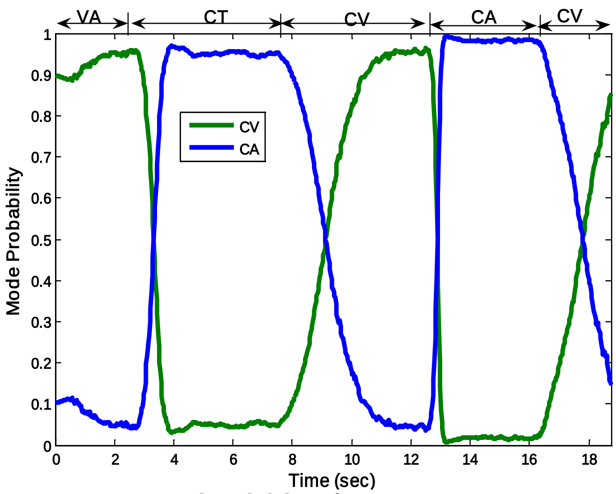

Fig-6a. Mode probabilities from IMMEKF-VA

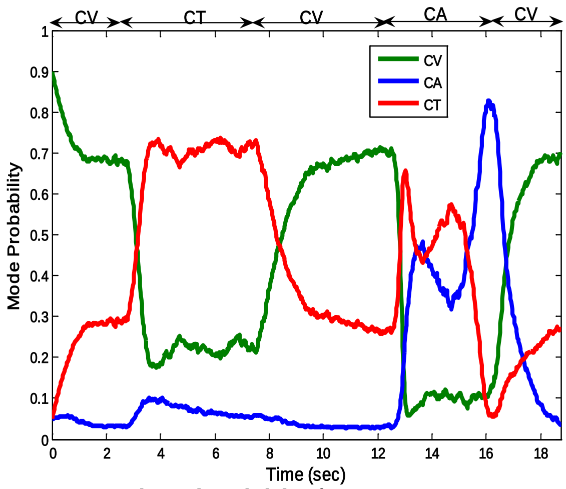

Fig-6b. Mode probability from IMMEKF-VAT